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TOOLS WITH ROTATIONAL MOTION FOR COMPLEX PROFILED HELICAL SURFACES MANUFACTURING PRODUCTS

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ABSTRACT

The paper recommends a general numerical method, for shaping the enveloping helical surfaces, based on the theory of surface generation by enveloping. The general contact equation can be personalised for all kinds of rotating motion generating schemes of the helical surfaces, the quotients a,b,c,d,e being a characteristic for the generating scheme.

1. Introduction

Compression is the most usual and The helical surfaces having complex profiles have known in the last twenty years a great and spectacular development such as technical applications in the field of screw pumps systems and of helical compression systems.

The operation of the mentioned helical systems is based on the sealling condition, checked along the meshing spatial line for a spindle pitch of the helical gear. The profile geometry of such helical gear imposes verification of two conditions:

- the" meshing law" ;
- the condition for the meshing plane line which has to be a closed curve in a plane perpendicular to the helical surfaces axis .

Remark: In this paper the terms "section profile" for the meshing curves profiles obtained as intersection of the helical surfaces by a plane which is perpendicular to the helical surface axis.

2. The profile geometry of the helical gear for screw compressors

Figure 1 presents the theoretical "section profile" that will be estimated for the profile of the rotational motion tools. The two axis reference systems choosen for the geometrical description of the section profile curves are:

-[x1,y1] having the reference point in '01' wflich is also the centre of male rotor, the 01 x 1 axis is the tooth profile axis of male rotor; [x2,y2] having the reference point in '02', which is also the centre of female rotor, the 02x2 axis is the empty space profile axis of female rotor .

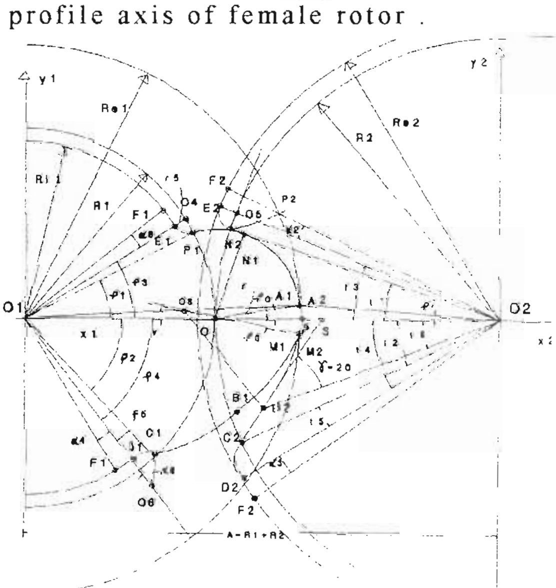

Fig.1.

The profile of male rotor

-The curve F101 = circle having the radius Ri 1

$x_1 = R_{i1} * \cos(\Phi_4 + \alpha_4 - \partial e);$

$y_1 = R_{i1} * \sin(\Phi_4 + \alpha_4 - \partial e)$

$\partial e \in [0, \alpha_4]$

$$t_1 = \sin(\Phi_4 + \alpha_4 - \partial e);$$
$$s_1 = -\cos(\Phi_4 + \alpha_4 - \partial e)$$

-The curve D1C1 = circle having the radius r_0= 06C1

$$x_1 = (R_{i1} + r_0) * \cos(\Phi_4) - r_0 * \cos(\Phi_4 + \partial e)$$
$$y_1 = -(R_{i1} + r_0) * \sin(\Phi_4) - r_0 * \sin(\Phi_4 + \partial e)$$
$$\partial e \in [0, \alpha_4]$$
$$t_1 = \sin(\Phi_4 + \partial e);$$
$$s_1 = -\cos(\Phi_4 + \partial e)$$

-The curve C1B1 enveloped by the straight line C2B2 from female rotor

$$x_1 = -R_2 * \cos(t_2 * (i+1)) +$$
$$+ \partial e * \cos(t_2 + y + \partial e_2 * (i+1)) + A * \cos(i * \partial e_2)$$
$$y_1 = -R_2 * \sin(t_2 * (i+1)) +$$
$$+ \partial e * \sin(t_2 + y + \partial e_2 * (i+1)) + A * \sin(i * \partial e_2)$$

$$t_1 = (i+1) * R_2 * \sin(t_2 + \partial e_2 * (i+1)) * \delta\partial e_2 / \delta\partial e -$$
$$- (i+1) * \delta\partial e_2 / \delta\partial *$$
$$* \sin(t_2 + \gamma + \partial e_2 * (i+1)) - i * A * \delta\partial e_2 / \delta\partial e *$$
$$* \sin(i * \partial e_2) + \cos(t_2 + \gamma + \partial e_2 (i+1))$$
$$s_1 = -(i+1) * R_2 * \cos(t_2 + \partial e_2 * (i+1)) * \delta\partial e_2 / \delta\partial e +$$
$$+ (i+1) * \delta\partial e_2 / \delta\partial e$$
$$* \cos(t_2 + \gamma + \partial e_2 * (i+1)) - i * A * \delta\partial e_2 / \delta\partial e *$$
$$* \cos(i * \partial e_2) + \sin(t_2 + \gamma + \partial e_2 (i+1))$$

$$\partial e_2 = ar\cos(k * (-r_2 * \cos(\gamma) + \partial e) / A) - \gamma - t_2$$
$$\partial e \in [0,1]$$

-The curve B1M1 enveloped by the circle M2B2 from the female rotor

$$x1 = -R_2 * \cos(\partial e_2 * (i+1)) +$$
$$+ (r - r_9) * \cos(\partial e_2 * (i+1) - \partial e_0) +$$
$$r_9 * \cos(\partial e_2 * (i+1) - \beta + \partial e - \partial e_0) + A * \cos(i * \partial e_2)$$
$$y1 = -R_2 * \sin(\partial e_2 * (i+1)) +$$
$$+ (r - r_9) * \sin(\partial e_2 * (i+1) - \partial e_0) +$$
$$+ r_9 * \sin(\partial e_2 * (i+1) - \beta + \partial e - \partial e_0) + A * \sin(i * \partial e_2)$$
$$t_1 = R_2 * (i+1) * \delta\partial e_2 / \delta\partial e * \sin(\partial e_2 * (i+1)) -$$
$$- \delta\partial e_2 / \delta\partial e * (r - r_9) * \sin(\partial e2 * (i+1) - \partial e_0) -$$
$$r_9 * (\delta\partial e_2 / \delta\partial e * (i+1) + 1) *$$
$$* \sin(\partial e_2 * (i+1) - \beta + \partial e - \partial e_0) -$$
$$- A * \delta\partial e_2 / \delta\partial e * i * \sin(i * \partial e_2)$$
$$s_1 = R_2 * (i+1) * \delta\partial e_2 / \delta\partial e * \cos(\partial e_2 * (i+1)) -$$
$$- \delta\partial e_2 / \delta\partial e * (r - r_9) * \cos(\partial e2 * (i+1) - \partial e_0) -$$
$$r_9 * (\delta\partial e_2 / \delta\partial e * (i+1) + 1) * \cos(\partial e_2 * (i+1) - \beta + \partial e - \partial e_0) -$$
$$- A * \delta\partial e_2 / \delta\partial e * i * \cos(i * \partial e_2)$$

$$\partial e_2 = \arcsin\left(k * \begin{pmatrix} (r_9 - r) * \sin(\partial e - \beta) - \\ - r_2 * \sin(\beta - \partial e + \partial e_0) \end{pmatrix} / A\right) +$$
$$+ \beta - \partial e + \partial e_0$$
$$\partial e \in [0, \beta]$$

-The curve M1A1 enveloped by the circle M2A2 from the female rotor

$$x_1 = -R_2 * \cos(\partial e * (i+1)) +$$
$$+ r * \cos(\partial e_2 * (i+1) - \partial e_0 + \partial e) + A * \cos(i * \partial e_2)$$
$$y_1 = -R_2 * \sin(\partial e * (i+1)) +$$
$$+ r * \sin(\partial e_2 * (i+1) - \partial e_0 + \partial e) + A * \sin(i * \partial e_2)$$
$$t_1 = R_2 * (i+1) * \delta\partial e_2 / \delta\partial e * \sin(\partial e_2 * (i+1)) -$$
$$- (\delta\partial e_2 / \delta\partial e * (i+1) + 1) * r * \sin(\partial e_2 * (i+1) - \partial e_0 + \partial e) -$$
$$- a * i * \delta\partial e_2 / \delta\partial e_2 * \sin(i * \partial e_2)$$
$$s_1 = -R_2 * (i+1) * \delta\partial e_2 / \delta\partial e * \cos(\partial e_2 * (i+1)) -$$
$$- (\delta\partial e_2 / \delta\partial e * (i+1) + 1) * r * \cos(\partial e_2 * (i+1) - \partial e_0 + \partial e) -$$
$$- a * i * \delta\partial e_2 / \delta\partial e_2 * \cos(i * \partial e_2)$$
$$\partial e_2 = -\arcsin(k * r_2 * \sin(\partial e_0 - \partial e) / A) - \partial e_0$$
$$\partial e \in [0, 2 * \partial e_0]$$

-The curve A1N1 shorted epicycloide

$$x_1 = A * \cos(i * \partial e) - r_4 * \cos(\partial e * (i+1) + t_1)$$
$$x_1 = -A * \sin(i * \partial e) + r_4 * \sin(\partial e * (i+1) + t_1)$$
$$t_1 = -A * i * \sin(i * \partial e) + r_4 * (i+1) \sin(\partial e * (i+1) + t_1)$$
$$s_1 = -A * i * \cos(i * \partial e) + r_4 * (i+1) \cos(\partial e * (i+1) + t_1)$$
$$\partial e \in [\partial e_{1\,max}, 0]$$

-The curve N1P1 enveloped by the straight line N2P2 from the female rotor efficient manufacturing procedure of rubber products.
It is mainly used on in technological design departments dealing with injection manufactured plastics.

$$x_1 = -(r_4 + \partial e) * \cos(t_1 - (i+1) * \partial e_2) + A * \cos(i * \partial e_2)$$
$$y_1 = (r_4 + \partial e) * \sin(t_1 - (i+1) * \partial e_2) + A * \sin(i * \partial e_2)$$
$$t_1 = -(r_4 + \partial e) * \sin(t_1 - (i+1) * \partial e_2) * (i+1) *$$
$$* \delta\partial e_2 / \delta\partial e - A * \sin(i * \partial e_2) * i \delta\partial e_2 / \delta\partial e -$$
$$- \cos(t_v - \partial e_2 * (i+1))$$
$$s_1 = -(r_4 + \partial e) * \cos(t_1 - (i+1) * \partial e_2) * (i+1) *$$
$$* \delta\partial e_2 / \delta\partial e + A * \cos(i * \partial e_2) * i \delta\partial e_2 / \delta\partial e -$$
$$- \sin(t_v - \partial e_2 * (i+1))$$

$$\partial e_2 = -ar\cos(k * (r_4 + \partial e) / A) + t_1$$
$$\partial e \in [0, r_2 - r_4]$$

-The curve P1E1 circle with the radius r_5 and centre point O_4

$$x_1 = (r_5 + R_{i1})*\cos(\Phi_3) + r_5 * \sin(\Phi_1 - \partial e)$$
$$y_1 = (r_5 + R_{i1})*\sin(\Phi_3) - r_5 * \cos(\Phi_1 - \partial e)$$
$$t_1 = -r_5 * \cos(\Phi_1 - \partial e)$$
$$s_1 = -r_5 * \sin(\Phi_1 - \partial e)$$
$$\partial e \in [0, \pi/2 + \Phi_1 - \Phi_3]$$

-The curve E1F1 circle with the radius R_{i1}

$$x_1 = R_{i1} * \cos(\Phi_3 + \partial e); \quad y_1 = R_{i1} * \sin(\Phi_3 + \partial e)$$
$$t_1 = -R_{i1} * \sin(\Phi_3 + \partial e); \quad s_1 = R_{i1} * \cos(\Phi_3 + \partial e)$$
$$\partial e \in [0, \alpha_6]$$

The profile of the female rotor

-The curve F2D2 = circle having the radius R_{i2}

$$x_2 = -R_{i2} * \cos(t_4 + \alpha_3 - \partial e);$$
$$y_2 = R_{i2} * \sin(t_4 + \alpha_3 - \partial e)$$
$$t_2 = -\sin(t_4 + \alpha_3 - \partial e);$$
$$s_2 = -\cos(t_4 + \alpha_3 - \partial e)$$
$$\partial e \in [0, \alpha_3]$$

-The curve D2C2 enveloped by the circle D1C1 from the male rotor

$$x_2 = (R_{i1} + r_0)*\cos(\Phi_4 + k * \partial e_1) -$$
$$- r_0 * \cos(\Phi_4 + \partial e + k * \partial e_1) - A * \cos(\partial e_1 / i)$$
$$y_2 = -(R_{i1} + r_0)*\sin(\Phi_4 + k * \partial e_1) +$$
$$+ r_0 * \sin(\Phi_4 + \partial e + k * \partial e_1) + A * \sin(\partial e_1 / i)$$
$$t_2 = -k * \delta\partial e_1 / \delta\partial e * (R_{i1} + r_0)*\sin(\Phi_4 + k * \partial e_1) +$$
$$+ r_0 * \cos(\Phi_4 + \partial e + k * \partial e_1)*(1 + k * \delta\partial e_1 / \delta\partial e) +$$
$$+ A * \sin(\partial e_1 / i) * \delta\partial e_1 / \delta\partial e / i$$
$$s_2 = -k * \delta\partial e_1 / \delta\partial e * (R_{i1} + r_0)*\cos(\Phi_4 + k * \partial e_1) +$$
$$+ r_0 * \sin(\Phi_4 + \partial e + k * \partial e_1) *$$
$$* (1 + k * \delta\partial e_1 / \delta\partial e) + A * \cos(\partial e_1 / i) * \delta\partial e_1 / \delta\partial e / i$$
$$\partial e_1 = \arcsin(k * i * (R_{i1} + r_0) * \sin(\partial e) / A - \Phi_4 - \partial e)$$
$$\partial e \in [0, \alpha_5]$$

C2B2 = straight line

$$x_2 = -r_2 * \cos(t_2) + \partial e * \cos(t_2 + \gamma);$$
$$y_2 = -r_2 * \sin(t_2) + \partial e * \sin(t_2 + \gamma);$$
$$t_2 = \sin(t_2 + \gamma); \quad s_2 = \cos(t_2 + \gamma);$$
$$\partial e \in [0, 1]$$

-The curve B2M2 circle having the radius r_a and the center point O_3

$$x_2 = -r_2 - (r_a - r)*\cos(\Phi) + r_a * \cos(\beta - \partial e + \Phi)$$
$$y_2 = (r_a - r)*\sin(\Phi) - r_a * \sin(\beta - \partial e + \Phi)$$
$$t_2 = r_a * \sin(\beta - \partial e + \Phi); \quad s_2 = r_a * \cos(\beta - \partial e + \Phi)$$
$$\partial e \in [0, \beta]$$

The curve B2 PA2 circle with radius r and center point O

$$x_2 = -r_2 + r * \cos(\partial e); \quad y_2 = r * \sin(\partial e);$$
$$t_2 = -r * \sin(\partial e); \quad s_2 = r * \cos(\partial e)$$
$$\partial e \in [-\partial e_0, \partial e_0]$$

-The curve A2N2 epicycloide trajectory of the point A1

$$x_2 = -A * \cos(\partial e / i) + r_y * \cos(v + \partial e + \partial e / i)$$
$$y_2 = -A * \sin(\partial e / i) + r_y * \sin(v + \partial e + \partial e / i)$$
$$t_2 = -A / i * \sin(\partial e / i) - r_y * (1 + 1/i) * \sin(v + \partial e + \partial e / i)$$
$$s_2 = -A / i * \cos(\partial e / i) - r_y * (1 + 1/i) * \cos(v + \partial e + \partial e / i)$$
$$\partial e \in [0, \partial e_{1\max}]$$

N2P2 Straight line

$$x_2 = -(r_{N2} + \partial e)*\cos(t_1); \quad y_2 = (r_{N2} + \partial e)*\sin(t_1)$$
$$t_2 = -\cos(t_1); \quad s_2 = \sin(t_1)$$
$$\partial e \in [0, r_2 - r_{N2}]$$

The limits of the parameters used for describing the profile

Input data $i = z_1 / z_2 = 1{,}5$; A= distance between the axis; R_{e1}=male external radius; R_{e2}=female external radius; h_1= imposed asimetrical quotient, $\partial e_0 = 10°$;

$$r_1 = A/(i+1); \quad r_2 = i * r_1; \quad R_{i1} = A - R_{e2};$$
$$R_{i2} = A - R_{e1}; \quad k = 1 + 1/i;$$
$$r = r_2 - R_{i2}; \quad O_1A_1 = \sqrt{r_1^2 + r^2 + 2 * r * r_1 * \cos(\Phi_0)};$$
$$v = \arcsin(r * \sin(\Phi_0) / O_1A_1);$$
$$O_2A_2 = \sqrt{r_2^2 + r^2 - 2 * r * r_2 * \cos(\Phi_0)};$$
$$< A_2O_2P_2 = \Phi' = \arcsin(r * \sin(\Phi_0) / O_2A_2);$$

The following equations:

$$\frac{A}{\sin(v + k * \partial e_{1\max} + t_1)} = \frac{r_2 * \cos(t_1)}{\sin(v + \partial e_{1\max})} =$$
$$= \frac{O_1A_1}{\sin(t_1 + \partial e_{1\max})};$$

are solved for the unknowns $\partial e_{1\max}$ and t_1 and then

$$t_2 = \arcsin(\sin(t_1) / h_1); \quad \Phi_1 = i * t_1;$$
$$r_5 = (r_1^2 - R_{i1} * R_{i1}) / 2 / R_{i1};$$
$$\Phi_3 = \Phi_1 + \arcsin(r_5 / (R_{i1} + r_5));$$
$$t_3 = \Phi_3 / i;$$

$$r_0 = O_6C_1 = (r_1 * r_1 - R_{i1} * R_{i1})/2/(R_{i1} - r_1 * \sin(\gamma));$$
$$\Phi_5 = \arcsin(r_0 * \cos(\gamma)/R_{i1} + r_0);$$
$$t_5 = \Phi_5 / i;$$
$$O_0 = \sqrt{r^2 + r_2^2 / 2 * r * r_2 * \cos(t_2 - \Phi')} = O_2A_2 = O_2M_2$$
$$\Rightarrow \beta = \pi/2 - \Phi_0 - t_2 - y$$

$$O_8 = C_2M_2 = \sqrt{O_0^2 + r_2^2 - 2 * r * r_2 * \cos(t_2 - \Phi')}$$
$$Q_1 = O_2S = r_2 / \sin(\gamma)/\sin(\gamma + t_2);$$
$$Q_2 = SC_2 = r_2 * \sin(t_2)/\sin(\gamma + t_2)$$
$$Q_3 = OS = r_2 - Q_1;$$
$$Q_4 = M_1S = \sqrt{r^2 + Q_3^2 - 2 * r * Q_3 * \cos(\partial e_0)}$$
$$\beta_6 = \arcsin(r * \sin(\partial e_0)/Q_4);$$
$$Q_5 = M_1B_2 = Q_4 * \sin(\gamma + t_2 - \beta_6)/\sin(\beta/2)$$
$$r_9 = r_a = Q_5 / 2 / \sin(\beta/2);$$
$$l = Q_2 - Q_4 * \sin(\beta/2 + \gamma + t_2 - \beta_0)/\sin(\beta/2)$$
$$r_4 = r_2 * \cos(t_1); \quad \Phi_2 = i * t_2;$$
$$\Phi_4 = \Phi_2 + \Phi_5; \quad t_4 = t_2 + t_5;$$
$$\alpha_2 = (\pi/3 - t_4 - t_3); \quad \alpha_3 = \alpha_2;$$
$$\alpha_5 = \pi/2 - \gamma - \Phi_5; \quad \alpha_4, \alpha_6 = i * \alpha_3$$

3. Milling cutter profile

According to the generation scheme presented in the figure 2, it is written the equation of the coordinates turning from the three axis reference system attached to the helical surface XO_bYZ, to the three axis reference system xo_syz attacned to the rotational tool :

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{vmatrix} * \begin{vmatrix} X \\ Y \\ Z \end{vmatrix} - \begin{vmatrix} l \\ n \\ 0 \end{vmatrix} \qquad (1)$$

or detailed:

$$\begin{cases} x = X - l \\ y = Y * \cos(\alpha) + Z * \sin(\alpha) - n \\ z = -Y * \sin(\alpha) + Z * \cos(\alpha) \end{cases} \qquad (2)$$

It is imagined the following discretization method (fig.1 and fig. 3) for writting the parametrical equations of the helical surface .

Each current point P on "section profile" curve Γ describe a helix according to the equation:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} X_p \\ Y_p \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ p(\beta) \end{vmatrix} \qquad (3)$$

or to the following detailed expresion:

$$\begin{cases} X = X_p * \cos(\beta) - Y_p * \sin(\beta) \\ Y = Y_p * \sin(\beta) - Y_p * \cos(\beta) \\ Z = p * (\beta) \end{cases} \qquad (4)$$

The geometrical state of surfaces enveloping condition, $\overline{A} * (\overline{n} \times \overline{r})$becomes in the reference system xyz

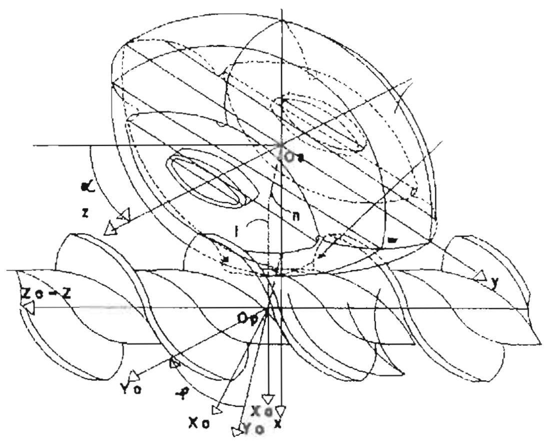

Fig.2.

$$\begin{vmatrix} 0 & 0 & 1 \\ n_x & n_y & n_z \\ x & y & z \end{vmatrix} = 0 \Rightarrow n_x * y - n_y * x = 0$$

(5)

With the help of figure 3 it is estimated the vector which is perpendicular to the helical surface E (in the reference system attached to the surface) as the result of vectorial multiplication of the tangent vector for the ordinary helix and the tangent vector for "section profile" curve (for a current point on this curve).

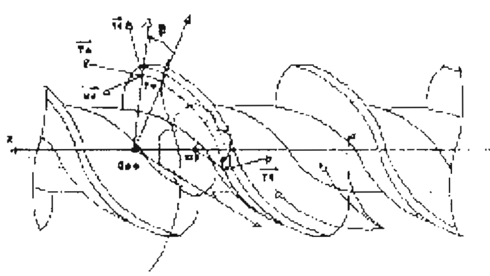

Fig.3.

The resulting perpendicular vector is rotated with the enveloping contact angle β which has to be solved.

$$\bar{N}_E = \bar{t}_e \times \bar{t}_{fr}$$

$\bar{t}_e$, which is the tangent vector for the ordinary helix was obtained from the equation (4) by differentiation using the parameter β. The tangent vector for the "section profile" curve is written using the "section profile" discretization (the equations from the second paragraph) and the rotating turn along oZ axis with the angle β. It can be stated know:

$$\bar{t}_e = \partial X/\partial\beta * \bar{i} + \partial Y/\partial\beta * \bar{j} + \partial Z/\partial\beta = = \begin{pmatrix} -X_p * \sin(\beta) - Y_p * \cos(\beta) * \bar{i} + \\ +(X_p * \cos(\beta) - Y_p * \sin(\beta)) \end{pmatrix} * \bar{j} + p * \bar{k} \quad (6)$$

$$\bar{t}_f = \partial X_p/\partial f * \bar{i} + \partial Y_p/\partial f * \bar{j} + 0 * \bar{k} = = T * \bar{i} + S * \bar{j} + 0 * \bar{k} \quad (7)$$

$$\bar{t}_{fr} = T * \cos(\beta) - S * \sin(\beta) * \bar{i} + +(T * \sin(\beta) + \cos(\beta)) * \bar{j} + 0 * \bar{k} \quad (8)$$

$$\begin{vmatrix} N_{Ex} \\ N_{Ey} \\ N_{Ez} \end{vmatrix} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \cos(\beta) - S\sin(\beta) & T*\sin(\beta) + S*\cos(\beta) & 0 \\ -X_p*\sin(\beta) - Y_p*\cos(\beta) & X_p*\cos(\beta) - Y_p*\sin(\beta) & p \end{vmatrix}$$

or

$$\begin{vmatrix} N_{Ex} \\ N_{Ey} \\ N_{Ez} \end{vmatrix} = \begin{vmatrix} p*(T*\sin(\beta) + S*\cos(\beta)) \\ -p*(T*\cos(\beta) - S*\sin(\beta)) \\ T*X_p + S*Y_p \end{vmatrix} \quad (9)$$

This vector turned to the reference system xyz give the vector n

$$\begin{vmatrix} n_x \\ n_y \\ n_z \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{vmatrix} * \begin{vmatrix} N_x \\ N_y \\ N_z \end{vmatrix} \quad (10)$$

or detailed:

$$\begin{cases} n_x = N_x = p*(T*\sin(\beta) + S*\cos(\beta)) \\ n_y = N_y*\cos(\alpha) + N_z*\sin(\alpha) \\ n = N_y*\sin(\alpha) + N_z*\cos(\alpha) \end{cases}$$

$$\begin{cases} n_x = p*(T*\sin(\beta) + S*\cos(\beta)) \\ n_y = -p*(T*\sin(\beta) + S*\cos(\beta)) + \\ +(T*X_p + Y_p*S)*\sin(\alpha) \\ n_z = p*(T*\sin(\beta) + S*\cos(\beta)) + \\ +(T*X_p + S*Y_p)*\cos(\alpha) \end{cases} \quad (11)$$

From (2) and (4) it can be stated:

$$\begin{cases} x = X_p*\cos(\beta) - Y_p*\sin(\beta) - l \\ y = \cos(\alpha)*(X_p*\sin(\beta) + Y_p*\cos(\beta)) + \\ +p*\beta*\sin(\alpha) - n \\ z = -\sin(\alpha)*(X_p*\sin(\beta) + Y_p*\cos(\beta)) + \\ +p*\beta*\cos(\alpha) \end{cases} \quad (12)$$

Replacing (12) and (11) in (6) it will be obtained the contact enveloping equation as:

$$\sin(\beta)*\begin{bmatrix} Y_p*\sin(\alpha)*(X_p*T + Y_pS) + \\ +p*l*S*\cos(\alpha) - p*n*T \end{bmatrix} + p^2*\beta*\sin(\beta)*T\sin(\alpha)$$

$$\cos(\beta)*\begin{bmatrix} -X_p*\sin(\alpha)*(X_p*T + Y_pS) - \\ -p*l*S*\cos(\alpha) - p*n*T \end{bmatrix} + p^2*\beta*\cos(\beta)*T\sin(\alpha) + +(p*\cos(\alpha) + A*\sin(\alpha))*(X_p*T + Y_p*S) = 0 \quad (13)$$

Considering the centering error m=0 and using the remark that l= A that is the distance between the rotor axis and the tool axis this equation becomes:

$$\sin(\beta)*\begin{bmatrix} Y_p*\sin(\alpha)*(X_p*T + Y_pS) + \\ +p*A*S*\cos(\alpha) - p*n*T \end{bmatrix} + p^2*\beta*\sin(\beta)*T\sin(\alpha)$$

$$\cos(\beta)*\begin{bmatrix} -X_p*\sin(\alpha)*(X_p*T + Y_pS) - \\ -p*A*S*\cos(\alpha) - p*n*T \end{bmatrix} + p^2*\beta*\cos(\beta)*T\sin(\alpha) + (p*\cos(\alpha) + A*\sin(\alpha))* *(X_p*T + Y_p*S) = 0 \quad (14)$$

The numerical application was developed according to the following alghoritm :

1) a number of "N" points on the "section profile' curve are known by their linear coordinates and by the tangent coordinates for each one of these points.

2) A = minimum radius of the "section profile" curve + the choosed radius of the tool, and the position angle of the tool α, are known;
3) the equation (14) is solved "N" times obtaining "N" values for the contact angle β;
4)each of the above mentioned values of the contact angle P cause a point of the enveloping contact line to be obtained with the equation (12);
5)each of these points (obtained from the equation 12) have a correspondin9 point in the axial section of the tool according to the figure 4 which details the scheme of axial tool profile projection by rotating the points of step 4.

$$x_{II} = \sqrt{x^2 + y^2}; \quad y_{II} = z$$

6)Using the .equations (11) the axis projections for the vector that is perpendicular to the helical surface in the contact point were computed. This perpendicular vector IS subject for the same projection by rotation (step 5). According to the geometrical condition of enveloping contact the vector crosses the rotating axis (axis of tool figure 4).

$$n_{xII} = \sqrt{n_x^2 + n_y^2}; \quad n_{yII} = n_z \qquad (16)$$

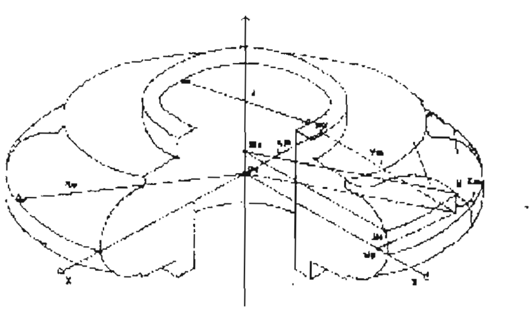

Fig.4.

The coordinates given by the equations (16) can be used for modelling the reversed envelopping and for checking pattern calculus.

4. The whirlpool tool

This type of tool is set in the same way as the milling cutter tool, the only diference is that contact line is positioned out of the space between the rotor axis and the tool axis. Therefore it can be stated that all the mathematical method presented in the paragraph 3 remains the same on except for the expresion and sign of A. Thus $A = R_{tool} - R_{irotor}$ (17) and for all the equations A has the minus sign .According to the above remarks the contact equation becomes:

$$\sin(\beta)*\begin{bmatrix} Y_p*\sin(\alpha)(X_p*t+Y_p*S)- \\ -p*A*S*\cos(\alpha) \end{bmatrix}+$$
$$+p^2*\beta*T*\sin(\beta)*\sin(\alpha)+$$
$$\cos(\beta)*\begin{bmatrix} -X_p*\sin(\alpha)(X_p*t+Y_p*S)+ \\ +p*A*T*\cos(\alpha) \end{bmatrix}+$$
$$+p^2*\beta*S*\cos(\beta)*\sin(\alpha)+$$
$$(p*\cos(\alpha)-A*\sin(\alpha))*(X_p*T+Y_p*S)=0 \qquad (18)$$

and the equation (12) become:

$$\begin{cases} x = X_p*\cos(\beta)-Y_p*\sin(\beta)+Al \\ y=\cos(\alpha)*(X_p*\sin(\beta)+Y_p*\cos(\beta))+ \\ +p*\beta*\sin(\alpha) \\ z=-\sin(\alpha)*(X_p*\sin(\beta)+Y_p*\cos(\beta))+ \\ +p*\beta*\cos(\alpha) \end{cases} \qquad (19)$$

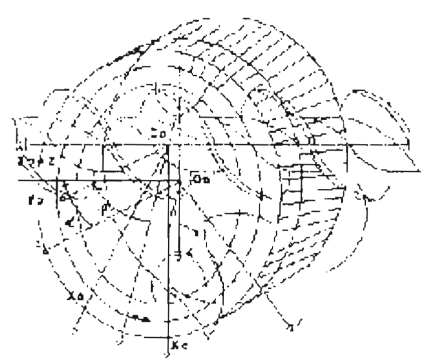

Fig.5.

The numerical alghoritm retains the same principles and steps.

5. Numerical result

On the basis of the equation described in paragraph 3 and 4 were obtain the following result representing the axial shape of the milling cutter and of the whirlpool tools for compressor rotors machining having the " section profile" curve like as described in figure 1.

Tab. 1.

Milling cutter profile for male

i	x tool	y tool	t tool	s tool
1	-73,3223	17,4953	-0,997483	-0,070899
2	-73,4030	14,9814	-0,980949	0,194260
3	-71,2791	15,5464	-0,576128	0,817358
4	-77,4312	11,5535	-0,510330	0,859978
5	-83,4347	8,3185	-0,435458	0,900208
6	-88,3481	6,1693	-0,363940	0,931422
7	-92,0833	4,0002	-0,277909	0,961422
8	-96,513	4,0141	-0,000000	1,000000
9	-99,1957	3,0968	-0,637388	0,770542
10	-100,0045	1,4027	-0,999979	-0,006422
11	-100,0005	0,4675	-0,999997	-0,002141
12	-100,0003	0,3896	-0,999998	0,001784
13	-100,004	-1,3247	-0,999981	0,006066
14	-99,3998	-3,2715	-0,797336	-0,603534
15	-96,5075	-5,0517	-0,254441	0,967088
16	-92,6903	-6,0517	-0,400780	-0,916174
17	-88,0632	-8,8328	-0,518648	-0,854987
18	-83,5962	-11,9114	-0,612391	-0,790554
19	-78,2480	-17,0080	-0,781770	-0,623566

Tab. 3.

Whirlpool tool for male rotor

i	x tool	y tool	t tool	s tool
1	-277,7613	14,2945	-0,997931	0,064282
2	-277,0608	11,0897	-0,934805	0,355158
3	-269,8723	5,7449	-0,136557	0,990632
4	-265,7077	5,2025	-0,121982	0,992532
5	-261,9360	4,7605	-0,111680	0,993744
6	-259,0524	4,4390	-0,112761	0,993622
7	-256,5828	4,1779	-0,087225	0,966188
8	-253,5366	4,0164	-0,053188	0,998584
9	-250,7981	3,0916	-0,638859	0,769323
10	-250,0042	1,4462	-0,999983	0,005824
11	-250,0004	0,4820	-0,999998	0,001941
12	-250,0003	-0,4017	-0,999990	-0,001617
13	-250,0037	-1,3658	-0,999904	-0,005500
14	-250,6248	-3,2978	-0,791912	-0,610635
15	-253,6020	-5,0521	-0,213831	-0,977088
16	-257,0665	-5,9234	-0,272659	-0,962110
17	-261,1069	-7,2104	-0,333225	-0,942047
18	-265,1790	-8,7952	-0,391651	-0,920113
19	-271,2538	-12,0585	-0,581515	-0,813535

Tab. 2.

Milling cutter profile for female

i	x tool	y tool	t tool	S tool
1	-74,0396	25,6575	-0,994613	-0,103650
2	-73,9413	24,6965	-0,995003	-0,099836
3	-73,8999	23,3990	-0,986861	0,161550
4	-75,6644	20,9562	-0,558921	0,829220
5	-70,8650	19,6315	-0,342493	0,939520
6	-82,5318	18,1789	-0,393579	0,919290
7	-86,2633	16,4545	-0,445162	0,095449
8	-89,7456	14,5987	-0,502217	0,864741
9	-93,9554	11,5776	-0,664018	0,747715
10	-97,1573	8,0226	-0,817818	0,575475
11	-99,2362	4,0759	-0,939860	0,341557
12	-99,9827	0,6136	-0,998416	0,056255
13	-99,5696	-3,0472	-0,961611	-0,274415
14	-86,6343	-10,6110	-0,121245	-0,992622
15	-83,3418	-10,8957	-0,180723	-0,983533
16	-78,4775	-11,6244	-0,111143	-0,993804
17	-74,5762	-12,3663	-0,474446	-0,880284
18	-73,1242	-14,2595	-0,998576	-0,053346
19	-73,1732	-15,2394	-0,998079	0,061939

Tab. 4.

Whirlpool tool for female rotor

i	x scula	y scula	t scula	s scula
1	-278,6973	24,9552	0,993863	0,110612
2	-278,5946	24,0157	0,994302	0,106591
3	-278,3748	22,7495	0,934393	0,356243
4	-276,1051	20,7168	0,393037	0,919522
5	-272,5936	19,8460	0,238357	0,971177
6	-267,9673	10,3615	0,365605	0,930769
7	-263,1657	16,1299	0,473136	0,880989
8	-258,9076	13,5564	0,564230	0,825617
9	-254,9703	10,2755	0,715537	0,698574
10	-252,2278	6,8079	0,849371	0,527795
11	-250,5515	3,2209	0,951747	0,306800
12	-250,0116	0,4662	0,998746	0,050055
13	-250,2915	-2,3235	0,969637	-0,244545
14	-247,9873	-1,6318	0,542803	-0,839859
15	-263,8066	-8,7711	0,386514	-0,922283
16	-270,4673	-11,4346	0,178464	-0,983946
17	-276,8140	-12,1179	0,352129	-0,935951
18	-277,7208	-13,8344	0,904438	-0,175730
19	-277,7932	-14,7963	0,997794	-0,066372

Milling cutters profile

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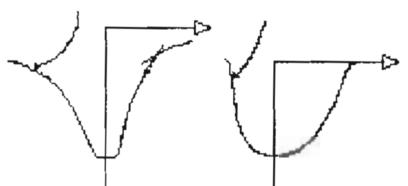

Whirlpool tools profiles

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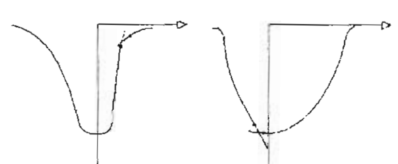

6. Conclusion

The alghoritms have a great generality so these can be used for any kind of "section profile" curve of any helical surface.

The contact equations 14 and 19 have the same followinq appearance:

$$a*\cos(\beta)+b*\sin(\beta)+c*\sin(\beta)+d*\cos(\beta)+e=0(*)$$

where the quotients a,b,c,d,e are according to the generating by enveloping scheme.

7. References

[1] **F. L. Litvin** - *New developments on theory of gearing.Eight world congress on the theory of machines and mechanisms*, Prague 1991;

[2] **Nicolae Oancea, s.a.** - *Metode numerice pentru profilarea sculelor* .M.E.I. 1992;

[3] **Y. Nakano** - *Method of calculating tool profiles for toothed workpieces having complicated tooth forms.* International symposium on gearing & Power transmisions, Tokyo 1991.

SCULE CU MIŞCARE DE ROTAŢIE PENTRU PRELUCRAREA SUPRAFEŢELOR ELICOIDALE CU PROFILE FRONTALE SPECIALE

(Rezumat)

În lucrare, se prezintă un algoritm, în baza unei modificări a condiţiei Nicolaev, pentru profilarea sculelor de tip disc (supafaţă de revoluţie, pentru generarea prin înfăşurare a melcilor pompelor şi compresoarelor, în scopul generalizării aplicării metodei.

Se prezintă exemple numerice pentru un profil uzual al compresoarelor elicoidale.

OUTILS AVEC LE MOUVEMENT DE ROTATION POUR LES PRODUITS HÉLICOÏDAUX PROFILÉS COMPLEXES DE FABRICATION DE SURFACES

(Résume)

Le papier recommande une méthode numérique générale, pour former les surfaces hélicoïdales d'enveloppement, basée sur la théorie de génération extérieure par l'enveloppement.

L'équation générale de contact peut être personnalisée pour toutes sortes de mouvement tournant produisant des arrangements des surfaces hélicoïdales, les quotients a, b, c, d, e étant une caractéristique pour l'arrangement se produisant.